

**Research of Stochastic Population Models Under Allee Effects
by Kolmogorov-Chapman Equation and WKB Approximation**

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In recent years, population models involving Allee effects have been extensively studied because of their current importance in the field of evolutionary biology. These studies of population dynamics actually transcend the limits of ecology and biology. This is because research on population dynamics is also useful in modeling a wide range of physical systems with chaotic properties. Allee effect in a population can be defined as a positive correlation between population density and mean individual fitness. If there is a critical value for population size value below which per capita population growth rate remains negative, this is the strong Allee effect. Also, when there is a positive correlation between population density and mean individual fitness but no critical value for population size value below which the per capita population growth rate remains negative, this is the weak Allee effect. It is possible to give stochastic properties to deterministic population models under the influence of Allee effects by adding in-population and out-of-population fluctuations, i.e. noise terms. A common method for calculating the average transition time from a steady state to extinction in populations with internal and external noise showing stochastic properties is to calculate the stationary probability distribution function of the population by the solution of Fokker-Planck equation. However, for populations under large fluctuations, the limits of the results obtained from the Fokker-Planck equation and alternative methods have not been studied sufficiently in the literature. In addition, we have come across alternative approaches in the literature, such as the calculation of the mean transition time by the Wentzel-Kramers-Brillouin (WKB) approximation method, which has been frequently used in the solution of the Schrodinger equation. In this study, we examine some stochastic population models under the influence of strong and weak Allee effects by using the master equation obtained by linearizing Kolmogorov-Chapman equation and using WKB approximation. We compare our results with those obtained from the solution of the Fokker-Planck equation and discuss the effects of this new approach on the average transition time.