Research of Stochastic Properties of a Population Model with Fokker Planck Equation Under Allee Effect

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Research on population models provides us rich theoretical tools for understanding the universal properties of systems that display chaotic behavior in different fields of science, as well as practical applications for population dynamics. For example, stability analysis of logistic population models have been an interesting research area for many years since it contributes to wide range of fields from evolutionary analysis to system biology, psychology to modeling of social events, chemical kinetics to kinetic theory of gases, modern cosmology to quantum chaos, and from artificial intelligence research to cryptology. In addition to this, recently, we see that, population models that describe populations with Allee effect have been extensively studied in the literature. It is expected that for Logistic population models in high densities, as the density of the population increases, per capita growth rate of the population decreases. With this usual result, in low densities, as the density of the population decreases, the decrease in the growth rate of population is known as Allee effect. If the per capita growth rate is negative when the population size falls below a certain threshold and the process results in extinction, this is strong Allee effect. However, if there is no such threshold as the population density decreases and the growth rate remains low but positive at low population density, this is weak Allee effect. In this study, firstly a population model under the influence of Allee effect will be discussed in terms of the limits of strong and weak Allee effect and the stability characteristics of the deterministic model by the means of its potential functions characterizing the population variation. Then, in-population and out-of-population fluctuations will be added to the population model to investigate their effects on the time evolution of population. In other words, noise terms whose statistical properties are clearly given will be added to the model. Thus, population change can be examined with a more realistic model considering stochastic properties. Stationary probability distribution function of population will be calculated by the solution of Fokker-Planck equation. In this way the effect of noise on stationary probability distribution function and the mean transition time between the steady states of the population will be discussed under strong or weak Allee effects for different parameters in the population model.